Is There Any Hope? Trapped Player in Row Call Game

QED 2022 Paper

by Noah Shiber

Junior Division (Grade 5)

Abstract

I introduce the game Row Call, including a -10 point penalty for the final trapped player. I investigate if it is still possible for a trapped player to win if I make changes to the original grid size or range of points given to each game cell. I discovered that when cells are worth the same amount of points, the trapped player always loses, no matter how many points are assigned. I also found the range needed for different grid sizes, with smaller grids usually needing a bigger range. For grids of 5 or above, only a range of 1-2 is needed.

Introduction

My math teacher showed me to a game called "Row Call," which was really fun to play. After playing it a few times, I started thinking about the different parts that make up the game and how they work together. I decided to investigate these parts by testing different changes to the game. I will describe what I found in this report. I begin by saying how the game works, then I set up a specific situation to test the effects of changing the original game's grid size and points per cell, to understand how these elements work together.

Body

The original game:

Roll Call happens in a 4x4 grid. Each cell on the grid is worth points from 1-10 (randomly chosen). The first table below is an example of an initial board (with cells filled by using a 10 sided-die). There are two players taking turns like in tic-tac-toe. The first player starts by choosing any number in the bottom row. After this first move, the rule is: players can choose any number in the same row or column of as the previous move (second table). But once a number is chosen, it blocks any choices past it. For example, if you choose the 10 on the top right corner, after the 9 next to it was chosen, you can only move down (third table).

8	8	9	10
5	1	3	9
10	3	2	3
2	1	4	2

8	8	9	10
5	1	3	9
10	3	2	3
2	1	↑ ←4→	2

8	8	9	←10 ↓
5	1	3	9
10	3	2	3
2	1	4	2

8	8	9	10
5	1	3	9
10	3	2	3
2	1	4	2

The game ends when a player is "trapped" and cannot make any further moves. The trapped player gets a penalty of -10 points. For example, on the 4th table above, if player A chose the bottom right 2 last, player B would have nowhere to go in their turn. The game is over and player B gets -10 points added to their score.

Initial questions:

In the original game, you can trap a player and still lose based on points. So I had to think about both the points and potential traps when I was playing. I wondered about how these parts interacted in the game. What would the game be like if all the cells gave the same points? Or what if their range of points was more limited? Would the points still be enough to beat the -10 points of a trap?

I began with the original 4x4 grid. I wanted to know if it was still possible to win when trapped if the points available had less weight. What would happen if all the all the numbers on the cells were the same, could the trapped player still win?

To keep the possibilities under control, I decided to focus only on scenarios where the board is as filled as possible, and the trapping only happens at the very end (no cases where the trapping happens before the board is fully explored). This scenario is a good one because it allows the players to collect more points.

When cells are worth the same amount (spoiler alert: the trapper always wins):

Let's say each cell is worth 1 point. Then the trapper would win as follows. Let's say player B is the trapper. Would they have an equal amount of turns or would player A get one more turn from starting? They would have an equal amount of turns because in this scenario B is the trapper, which means B has the last turn: A could not get another turn because they got trapped. So they have the same amount of turns, and if all the cells are worth 1, then they have the same amount of points. However, A got trapped and would thus lose 10 points and lose the game. The same would happen even if each cell was worth 1000 points: they both get an equal amount, but one would get trapped and lose 10 points.

But what if B got trapped? Then it would be the same except that A started and B got trapped so B couldn't do the last move. So A had one move extra, more than B. Again B has a minus 10, so B looses even worst when trapped if starting second.

This led me to the conclusion that if all the cells were the same, the trapper, whether A or B, would always win.

Restricted Ranges:

What about if each cell was not the same, but varied less than in the original game. Say have a range of 1-2, or 1-3, instead of 1-10.

Since I am trying to figure out if it is possible for the trapped player to win at all, I decided to stick to a very unlikely scenario where the trapped player is as lucky as possible. Imagine the trapped player always gets the maximum points per cell, and the trapper always gets the least points. And the trapped player gets as many turns as possible while still ending up trapped. We will see that sometimes, even under this maximum luck, it is still impossible for the trapped player to win.

Range 1-2: Player A gets 2 points on every turn, until they are trapped (for a total of 8 turns). So they get 16 points, minus the 10 points for being trapped, for a total of 6. Player B gets 8 turns, but only scores 1 per turn, for a total of 8. 6<8, so the trapper still wins even if player A is maximally lucky.

Range 1-3: Player A gets 3 points on every turn, until they are trapped (for a total of 8 turns). So they get 24 points, minus 10 points for being trapped, for a total of 14. Player B gets 8 as described above. 14 > 8. Therefore, it is possible for the trapped player to win on the original 4x4 game, if the cells range from 1-3.

So with the original 4x4 grid, we need a range of at least 1-3 to make it possible for a trapped player to win. What about if we made the board smaller? If the board is smaller, there are less points available for a potential trapped player to be able to beat the -10 points penalty. That makes it harder for the trapped player to win. So I suspected we might need a bigger range. This turned out to be correct, as we will see below.

Will the 1-2 range that worked for the 4x4 grid still work on a 3x3 grid? Let's see.

First let's consider how many turns a trapped player gets, now we have an odd number of cells on the grid. If it were even, player B could trap player A by simply selecting the last square available. Each player would get the same number of turns. But since the number of cells is odd, player B would have to trap player A one move before the last turn, which means they would both get 4 moves. The game ends with the trap, before the last square is selected.

3x3 Grid, 1-3 Range:

Trapped player gets 3 points on every turn, for four turns: 4x3 =12-10=2

Trapper gets 1 on every turn: 4x1 = 4

2<4, so the trapped player loses.

3x3 Grid, 1-4 Range:

Trapped player gets 4 on every turn: 4x4 = 16-10=6

Second player gets 1 on every turn: 4x1 = 4

6>4, so the trapped player wins!

So it is possible for a trapped player to win on a 3x3 board with a range of 1-4

This made me curious to see the pattern between board size and the range needed. In this case, the side of the grid was smaller by 1 (from 4X4 to 3X3) and had to increase the minimum range by 1 (from 1-3 to 1-4). Is that always the case? Here is what happens at different grid sizes:

A 1x1 is a bit of a weird scenario, but I wanted to think about it just to cover all scenarios. In this case, the first player always wins by immediately trapping the second player! The trapped player always loses, no matter the range.

For other calculations, here are the results (if you are curious, calculations are in the appendix):

Minimal range required for a trapped player to win per grid size

Grid Size	Range
2	1-7
3	1-4
4	1-3
5	1-2

In grids 6x6 or larger, a minimal range of 1-2 is enough to make a trapped player win possible.

So there is not a simple pattern where +1 on the side of the grid is equal to -1 on the top of the range.

Conclusion

Roughly speaking, the smaller the board, the bigger the range has to be to compensate for the -10 points and make a trapped player win possible. The limits are: a range of just 1 is never enough, no matter the size of the board (not considering negative points). And if the grid had just one cell (1x1), a trapped player could never win, regardless of what range is given.

Thinking about these unlikely scenarios was fun and allowed me to better understand the relationship between some key parts of this game: board size, cell points and trapped penalty.

Appendix

1. **Calculations:**

2X2 Grid

Range 1-4
Trapped gets 4 on every turn: 2x4 = 8-10=-2 (yikes, better of not playing!)
Second player gets 1 on every turn: 2x1 = 2
-2<2: Trapped player loses badly, even in "luckiest" scenario

Range 1-5
Trapped player gets 5 on every turn: 2x5 = 10-10= 0
Second player gets 1 on every turn: 2x1 = 2
0<2: Trapped player loses

Range 1-6
Trapped player gets 6 on every turn: 2x6 = 12-10= 2
Second player gets 1 on every turn: 2x1 = 2
Conclusion: with a range of 1-6 the player can hope for a tie

Range 1-7
Trapped player gets 6 on every turn: 2x7 = 14-10= 4
Second player gets 1 on every turn: 2x1 = 2
Conclusion: finally with a range of 1-7, the trapped player could win

5x5 Grid

Range 1-2
Trapped player Starts, gets 2 on every turn: 12x2 = 24-10=14
Second player gets 1 on every turn: 12x1 = 12
14>12: a trapped player can win
Conclusion: with a range of 1-2, the trapped player could win

1x1, 3x3 and 4x4 grids were discussed in the text, the remaining grids can be inferred from these results